FURTHER PROPERTIES OF IBNR CLAIMS RESERVING

Properties of IBNR claims reserving are discussed in De Actuaris, Bulletin of the Dutch Actuarial Society, January 2004 (in Dutch). The following is a supplement to that article.

1. ***Trends occur in three directions, but the directions are not independent***

A model is supposed to capture the trends in the data, so it is essential to consider the geometry of trends in the three directions: *development year*, *accident year* and *calendar year*.

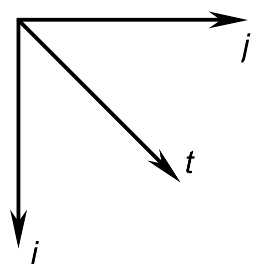

FIGURE 1

The calendar year variable t can be expressed as $t = i + j$. Trends in the development year and accident year directions are not projected onto each other. A trend in the calendar year direction is projected onto the development year and accident year directions and vice-versa.

To capture changing trends in all three directions, it is essential for the model to have the potential to adapt to those changes – to have the possibility of parameters in each of the three directions. Again, because of the linear dependence between directions, it is necessary to impose a restriction. Examination of real data indicates that exposure-adjusted payments do not generally have long-term trend in the accident year direction *after trends in calendar years are accounted for*. So a model need only capture long-terms trend in the other two directions.

2. ***Changing calendar year trends cannot be modelled with parameters in the other directions.***

Because of the relatedness of trends in the three directions, we cannot properly model the trends in the other two directions if we don't properly capture the changing trends in the calendar year direction. It is not sufficient to make some assumption about calendar years – the appropriateness of the model must be examined in the calendar year direction.

We cannot make informed judgement about the future if we cannot understand the past. Without looking at the calendar year direction, we *cannot* understand the past.

3. ***Calendar year effects are often the most important***

Calendar year effects do not go away in a continuous-time framework. Sometimes the rate of superimposed inflation is stable (with noise about a long term trend) for many years, but many times it is not. In either case it needs to be properly assessed. It is not sufficient to use the inflation in the payments to deflate the payments and then pretend the changing inflation was not present – the characteristics of the inflation process are central to forecasting the insurer's liabilities.

4. ***The standard deviation of the payment process is proportional to the mean***

The gamma distribution only satisfies the condition that the standard deviation is proportional to the mean if restrictions are placed on its parameters.

To see the effect graphically assume that a gamma distribution where the mean as a function of time is given by

$$\mu(t) = \alpha(t)\ \beta(t) = 0.\ 774\ exp(0.256\ t)\ ,$$

and further that $\alpha(t) = 100\ exp(0.256\ t)$ and $\beta(t) = 0.00774$.

Figure 2 below shows simulated values from this process on the original and logarithmic scales respectively. Note that the variability on the log scale decreases as the mean on the log scale increases. This applies when $\alpha(t)$ changes with t.

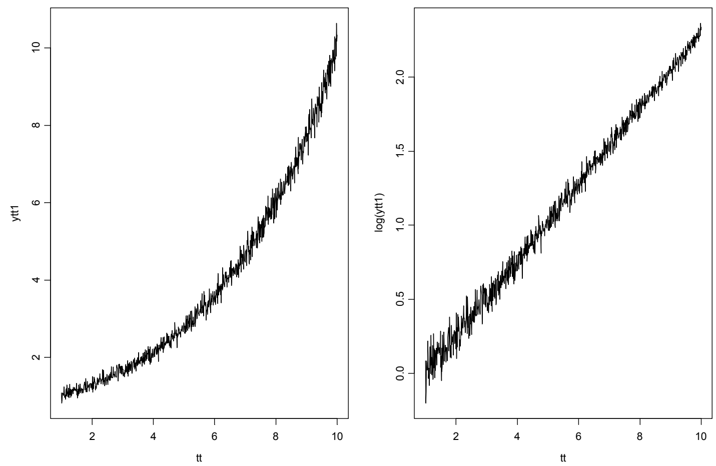

FIGURE 2

However, if we keep the same model for the mean, $\mu(t)$, but now assume that $\alpha(t) = 100$, $\beta(t) = 0.00774\ exp(0.256\ t)$ then we see

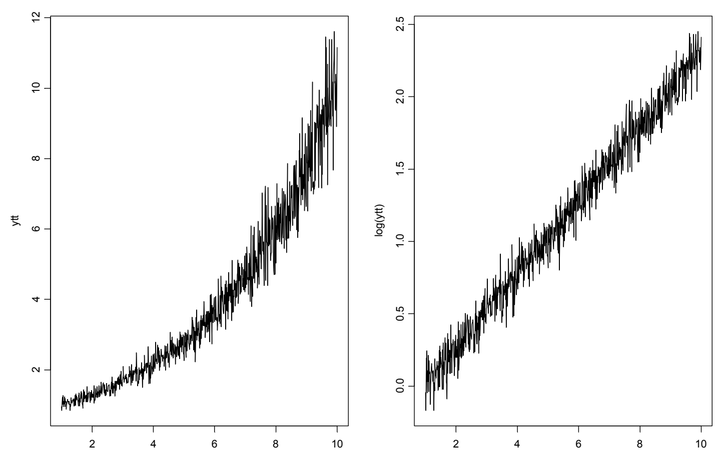

FIGURE 3

To get constant variability with a gamma distribution on a log scale we must have $\alpha(t)$ constant.

However for *any* lognormal distribution we see behaviour similar to figure 3 above; it automatically has the necessary property. Further, when one looks at the residuals from a gamma model (even where all trends are captured), the residuals for the development periods with the largest increments with noisy data can tend to be somewhat right-skewed. The residuals from a lognormal fit are generally quite symmetric, except occasionally for the very *smallest* payments; the fit to the distribution of errors is often better where it is most important.

A related point is that if the standard deviation is not proportional to the mean, the standard deviation as a percentage of the mean (the coefficient of variation) changes *if you change the units*. You might be surprised if someone claimed that you can change from pounds to euros, do the modelling and forecasting, and convert the mean and standard deviation back to pounds, and your answers are more accurate than if you'd done the analysis in pounds!

Two wrongs do not make a right

One must take care not to fix only one of the problems discussed here and in the de Actuaris paper and then blame the results on fixing the problem rather than on ignoring the remaining problems. The effects are not all independent! So, for example, if one uses a badly overparameterised model with a lognormal error term, one should expect to get a high answer *because* the model is badly overparameterised. Indeed if one does *not* get a very high answer with this bad model, the forecasting methodology is actually flawed. It is ludicrous to then blame the large result on the lognormal when the fault is overparameterisation. Using the lognormal is merely allowing you to see the problem; it is not the cause of the problem. If a new pair of spectacles allowed you to see that your car had badly scratched paintwork, you attend to the problem with the car – you do not put the old spectacles back on, even if you own several old pairs and the car looks okay through them all!

Some features we think forms the basis of good models

Parameters in the accident year direction determine the level from year to year; often the level (after adjusting for exposures) shows little change over many years, requiring only a few parameters. The parameters in the development year direction represent the trend from one development year to the next. This trend is often linear (on the log scale) across many of the later development years, generally requiring only one parameter to describe the tail of the development. The parameters in the calendar year direction describe the trend from calendar year to calendar year. If the original data are inflation-adjusted (by a price or wage index) before being transformed to the log scale, the calendar year parameters represent superimposed (social) inflation, which may be stable for many years or may not be stable at all. This is determined in the analysis. We see that very often only a few parameters are required to describe the trends in the data. Consequently, the best identified model for a particular loss development array is likely to be parsimonious. This allows us to have a clearer picture of what is happening in the incremental loss process, which is an important part of being informed about the business.